APPENDIX D-4. OIL-FIRED WATER HEATERS CONDUCTIVE HEAT LOSS CALCULATION

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This Appendix presents the detailed approach taken in this analysis to calculate the heat loss through the top and sides of oil-fired water heaters.

Water heater jacket heat losses are calculated as conductive heat losses from the water through the insulation to the jacket in series with convective and radiative heat transfers from the jacket to the surrounding air and environment. Conductive heat losses are calculated using standard conductive heat transfer equations. The equations for conductive heat transfer through the top and sides are:

$$Q_{cond\_top} = (k \cdot area_{top} \ / \ th_{insul}) \cdot (T_{tank} - T_{jacket}) \qquad \textbf{(Eq. D-4.1)}$$

and

$$Q_{cond\_side} = [(k \cdot 2 \cdot pi \cdot height) \ / \ (144 \cdot \ln \ (diam_{jacket} \ / \ diam_{tank}))] \cdot (T_{tank} - T_{jacket}) \qquad \textbf{(Eq. D-4.2)}$$

where

k = equivalent conductivity of insulation (Btu-in/hr-ft^2-ºF) or (W/m^2-ºC)
$area_{top}$ = area of top of tank (ft^2) or (cm^2)
th_{insul} = thickness of insulation (in) or (cm)
$height$ = height of tank (in) or (cm)
$diam_{jacket}$ = diameter of jacket (in) or (cm)
$diam_{tank}$ = diameter of tank (in) or (cm)
T_{tank} = temperature of tank (ºF) or (ºC)
T_{jacket} = temperature of jacket (ºF) or (ºC)
144 = conversion factor (ft^2/in^2) (non-SI units only)

The foam insulation is HFC-245fa, with a conductivity (k) of 0.144 Btu-in/hr-ft^2-ºF (0.027 W/m^2-ºK).[1]

Convective heat loss from the jacket to the surrounding air is calculated as:

$$Q\_conv = \Sigma_i \ hconv_i \cdot area_i \cdot (T_{jacket} - T_{amb}) \qquad \textbf{(Eq. D-4.3)}$$

where

$hconv_i$ = convective heat transfer coefficient for surface i (Btu/hr-ft^2-ºF) or (W/m^2-ºC),
$area_i$ = area of surface i of tank (ft^2) or (cm^2),
T_{amb} = temperature of ambient air (ºF) or (ºC).

The convective heat transfer coefficient varies depending on surface orientation, so the total convective heat loss is a summation of all of the heat losses from the tank surfaces exposed to the surroundings (the top and sides of the tank). The relationships used for the convective heat transfer coefficients are:

for horizontal heat flow:	$hconv = 0.19\ (T_{jacket}\text{-}T_{amb})^{0.33}$	(Btu/h-ft^2-ºF)
for vertical upwards heat flow:	$hconv = 0.22\ (T_{jacket}\text{-}T_{amb})^{0.33}$	(Btu/h-ft^2-ºF)

or

for horizontal heat flow:	$hconv = 1.31\ (T_{jacket}\text{-}T_{amb})^{0.33}$	(W/m-ºC)
for vertical upwards heat flow:	$hconv = 1.52\ (T_{jacket}\text{-}T_{amb})^{0.33}$	(W/m-ºC)

The radiative heat loss from the jacket to the surroundings is:

$$Q_{rad} = 3_i\ h_{rad} \cdot area_i \cdot (T_{jacket} - T_{amb}) \qquad \textbf{(Eq. D-4.4)}$$

where the radiative heat transfer coefficient is defined as:

$$h_{rad} = 0 * F * [(T_{jacket\_R})^2 + (T_{amb\_R})^2] \cdot (T_{jacket\_R} + T_{amb\_R}) \qquad \textbf{(Eq. D-4.5)}$$

where

F	= the Stephan-Boltzman constant, 0.1714 * 10$^{\{-8\}}$ Btu/hr-ft^2-ºR^4 (5.6697 x 10^{-8} W/m^2-K^4)
0	= emissivity of water heater surface, or white painted surfaces, 0.87
T_{jacket_R}	= the absolute temperature of the water heater jacket (ºR) or (K)
T_{amb_R}	= the absolute temperature of the environment around the water heater, (ºR) or (K)

Eq. D-4.5 is based on the simplifying assumption that the jacket is completely surrounded by a black body enclosure at the ambient temperature.[2] Since in most cases the space the tank is located in is significantly larger than the tank itself, this is a good working assumption.

At steady-state conditions, the heat flow from the tank through the insulation must equal the heat leaving the jacket by convective and radiative cooling.

$$Q_{cond} = Q_{conv} + Q_{rad} \qquad \textbf{(Eq. D-4.6)}$$

All three terms are dependent on the unknown jacket temperature, with the radiative and convective heat losses non-linear with temperature. To solve equation (6), the jacket temperature is chosen iteratively until the conductive heat loss matches the sum of the convective and radiative losses. This jacket temperature can then be used to solve for the total conductive heat loss from the top and sides of the tank.

Both the recovery efficiency (RE) and the standby heat loss coefficient (UA) are changed by a reduction in total jacket heat loss, as it occurs continuously. Since jacket losses are continuous losses, they are related to the RE through Eq. D-4.7 and the losses when the water heater is firing are a fraction of rated input, which does not go directly into heating water:

$$Loss\_continuous + Loss\_flue\_on = P_{on} \cdot (1 - RE) \qquad \textbf{(Eq. D-4.7)}$$

where P_{on} = rated input power

Thus,

$$RE = 1 - (Loss\_continuous + Loss\_flue\_on) / P_{on} \qquad \textbf{(Eq. D-4.8)}$$

And subtracting the RE for two different levels of jacket loss when all other losses are held constant gives:

$$RE_2 - RE_1 = (Loss\_continuous_1 - Loss\_continuous_2) / P_{on} \qquad \textbf{(Eq. D-4.9)}$$

or

$$RE_2 - RE_1 = (Jacket\_loss_1 - Jacket\_loss_2) / P_{on} \qquad \textbf{(Eq. D-4.10)}$$

The standby heat loss rate is:

$$Standby\ heat\ loss\ rate = UA\ (T_{tank} - T_{amb}) \qquad \textbf{(Eq. D-4.11)}$$

This "standby heat loss rate" is the average rate of heat input needed to maintain a tank at a constant temperature during standby. It includes continuous losses, flue losses when firing, and flue losses and piping losses during the off-cycle. An overall energy balance on the standby period yields:

$$UA \cdot (T_{tank} - T_{amb}) \cdot (24 - BOH_{draw}) = (Loss\_continuous + Loss\_flue\_off) \cdot (24 - BOH_{st}) + BOH_{st} \cdot P_{on} \cdot (1 - RE) \qquad \textbf{(Eq. D-4.12)}$$

where

BOH_{draw} = Burner operating time to make up for hot water drawn from the tank (hr),

BOH_{st} = Burner operating time to make up for standby losses (hr).

BOH_{st} can be calculated from the total energy required during the standby period divided by the energy input during firing. The energy required to make up for losses during the standby period is equal to the energy input for the whole DOE test minus the energy that is actually used to heat water removed from the tank. The procedure to develop the equation for BOH_{st} is:

$$Q_{stby} = Q_{in} - Q_{recov} \qquad \textbf{(Eq. D-4.13a)}$$

$$Q_{in} = Q_{draw} / EF \qquad \textbf{(Eq. D-4.13b)}$$

where EF = energy factor

$$Q_{recov} = Q_{draw} / RE \qquad \textbf{(Eq. D-4.13c)}$$

$$Q_{stby} = BOH_{st} \cdot P_{on} = Q_{draw} / EF - Q_{draw} / RE \qquad \textbf{(Eq. D-4.13d)}$$

and

$$BOH_{st} = 1/ P_{on} \cdot Q_{draw} \cdot (1/EF-1/RE) \qquad \textbf{(Eq. D-4.13e)}$$

And by default

$$BOH_{draw} = BOH\text{-}BOH_{st} \qquad \textbf{(Eq. D-4.14)}$$

Inserting Eq. D-4.12 in Eq. D-4.13 and solving for the loss components gives:

$$Loss_{continuous} + Loss_{flue\_off} = \frac{UA \cdot (T_{tank} - T_{amb}) \cdot (24 - BOH + BOH_{st}) - Q_{draw} \cdot (1 - RE) \cdot \left(\frac{1}{EF} - \frac{1}{RE} \right)}{24 - BOH} \qquad \textbf{(Eq. D-4.15)}$$

By substituting in Eq. D-4-15 the known parameters from the analytic baseline water heater, it is possible to calculate the combined *Loss_continuous* + *Loss_flue_off* values for any of the design options.

REFERENCES

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